SOCP based Robust Detector for Cooperative Spectrum Sensing in MIMO Cognitive Radio

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***Abstract*—In this paper we present a robust detection scheme for cooperative spectrum sensing in cognitive radio (CR) networks with channel uncertainties. We consider a soft-decision scenario at the fusion center for primary user detection, based on the sensed test statistics submitted by the cooperating secondary users. The scheme presented models the channel state information (CSI) uncertainty employing an ellipsoidal uncertainty set. It is then demonstrated that the optimal linear discriminator for cooperative spectrum sensing towards primary user detection in a CR system can be formulated as a second order cone program (SOCP). Further, we also formulate a relaxed robust detector (RRD) and a multicriterion robust detector (MRD) that maximally separate the hypothesis ellipsoids at low signal-to-noise ratio (SNR) and deep fade conditions. Simulation results demonstrate that the detection error performance of the proposed CSI uncertainty aware robust spectrum sensing schemes are significantly lower compared to other uncertainty agnostic schemes.**

I. Introduction

The emerging broadband wireless technologies have led to a tremendous rise in demand for radio spectrum as they require a large amount of bandwidth to support multimedia applications. This growth has outstripped spectrum availability, leading to enormous pressure and scarcity of available spectrum. Studies in [1], [2] reveal that the spectrum of licensed/primary users (PUs) is highly under-utilized. To improve the efficiency of spectrum utilization, CR has been proposed a revolutionary new paradigm to allow a set of secondary users (SUs) to opportunistically access vacant spectrum bands. Hence, one of the key processes of a CR terminal is to reliably sense the spectrum for absence of the licensed PU prior to such transmission.

In this context, several spectrum sensing techniques have been proposed in literature [3]–[5]. They can be broadly classified as being either local or cooperative in nature. It has been shown that cooperative sensing, in which a fusion center combines the sensing results from several cooperating users, has a much higher reliability for PU detection compared to local schemes. Soft-decision schemes such as maximal ratio combining (MRC) [6], which has the lowest detection error, requires perfect channel estimation. Obtaining CSI is a significantly challenging task in a wireless communication system. Moreover, it is even more challenging in a cooperative scenario involving multiple users. Hence, realistically, it is possible to only obtain approximate channel estimates due to errors in channel estimation combined with the limited feedback in wireless channels.

Hence, for such scenarios, we propose a cooperative spectrum sensing scheme incorporating uncertainty in the available CSI. The channel ambiguity is modeled as an ellipsoidal uncertainty set centered at the estimate of the multiuser channel coefficient vector, which constitutes the nominal CSI. We then propose a linear detector to minimize the worst case spectrum sensing error amongst these ellipsoidal uncertainty sets. It is demonstrated that the above problem can be formulated as a SOCP. Being convex in nature, the optimal solution can be computed with a high degree of reliability. We then propose the relaxed robust and multicriterion detectors and compare them with the conventional uncertainty agnostic detector.

The rest of the paper is organized as follows. Section II describes the scenario of cooperative spectrum sensing considered in this paper followed by the multiuser multiple-input multiple-output (MIMO) cognitive radio wireless system model and the uncertainty aspects of the channel. Section III describes the proposed cooperative spectrum sensing techniques and formulates the framework for computation of the robust detector, RRD and MRD. Simulation results are presented in section IV and we conclude in section V.

II. System Model

Consider a CR network with a PU base station, N SUs and a fusion center. We consider a MIMO wireless CR system in which the PU base-station possesses N_t transmit antennas while each SU has N_r receive antennas. Hence, the baseband system model of the above multiuser MIMO system for the n^{th} transmitted symbol vector is given as,

$$\mathbf{y}_i(n) = \mathbf{H}_i\mathbf{x}\left(n\right) + \eta_i\left(n\right) \tag{1}$$

where $\mathbf{y}_i(n) \in \mathbb{C}^{N_r \times 1}$ is the received N_r dimensional signal vector at the i^{th} SU corresponding to the PU base station broadcast symbol vector $\mathbf{x}\left(n\right) \in \mathbb{C}^{N_t \times 1}$. The vector $\boldsymbol{\eta}_i(n) \in \mathbb{C}^{N_r \times 1}$ of i^{th} SU at time instant n, is additive spatio-temporally white Gaussian noise with covariance $\mathrm{E}\left\{\eta_i(n)\eta_i(n)^H\right\} = \sigma^2\mathbf{I}_{N_r}$. The matrix $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix corresponding to the channel between the PU base station and i^{th} SU. Each of the elements

$h_i(r,t)$ of the matrix $\mathbf{H}_i$ denotes the fading channel coefficient between the t^{th} transmit antenna of the PU base-station and the r^{th}receive antenna of the i^{th} SU. Let the channel matrices corresponding to the N cooperating SUs be stacked as $\mathbb{H} \in \mathbb{C}^{NN_r \times N_t}$, given as,

$$\mathbb{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_N \end{bmatrix}$$

We consider the transmission of a beacon signal by the PU base station indicating the presence or absence of the PU. Let the beacons $\mathbf{p}_b, -\mathbf{p}_b \in \mathbb{C}^{N_t \times 1}$ denote the presence and absence of the PU respectively. For instance, the canonical signal $\mathbf{p}_b = [1,1,...,1]^T$ can be employed as a possible beacon. The theory proposed below however is not restricted to such antipodal beacon sets and can be extended in a relatively straight forward manner to other beacon signals. Each SU senses the PU base-station beacon and conveys the information to the fusion center. On receiving the measurements from all the cooperating SUs, the fusion center collates these statistics towards PU presence or spectral hole detection in the CR system. From the system model described above, the concatenated fusion center signal $\mathbf{y}_f(n) \in \mathbb{C}^{NN_r \times 1}$ can be described as,

$$\mathbf{y}_f(n) = \mathbb{H}\mathbf{x}(n) + \eta_f(n),$$

where $\eta_f(n) \triangleq \left[\eta_1(n)^T, \eta_2(n)^T, ..., \eta_N(n)^T\right]^T$ denotes the concatenated receiver noise vectors. Let the vector $\mathbf{h} \in \mathbb{C}^{NN_r \times 1}$ be defined as $\mathbf{h} = \mathbb{H}\mathbf{p}_b$. Hence, the PU detection problem can be formulated as the binary hypothesis testing scenario, with the null hypothesis $\mathcal{H}_0$ and alternative hypothesis $\mathcal{H}_1$ denoting the absence and presence of the primary user respectively. This can be in turn described as,

$$\mathcal{H}_0 : \mathbf{y}_f(n) = -\mathbf{h} + \eta_f(n) \quad (2)$$
$$\mathcal{H}_1 : \mathbf{y}_f(n) = \mathbf{h} + \eta_f(n) \quad (3)$$

It is well known [7] that the optimal detection error minimizing detector for the additive white Gaussian noise scenario above is the matched filter detector described as,

$$\mathcal{H}_0 : \mathbf{w}^H\mathbf{y}_f(n) < 0 \quad (4)$$
$$\mathcal{H}_1 : \mathbf{w}^H\mathbf{y}_f(n) \geq 0, \quad (5)$$

where $\mathbf{w} \triangleq \frac{1}{\|\mathbf{h}\|}\mathbf{h}$ is the spatial filter matched to the concatenated array response vector $\mathbf{h}$. In fact, $\mathbf{w}$ is the normal to the optimal separating hyperplane corresponding to the decision regions $\mathcal{H}_0$, $\mathcal{H}_1$ as shown in Fig.1. Recent research in convex optimization has resulted in the development of powerful techniques for computation of optimal linear classifiers. Hence, the above decision problem of computing the optimal separating hyperplane can be formulated as,

$$\begin{aligned} \min. \ & \|\mathbf{w}\|_2 \\ \text{s.t.} \ & \mathbf{w}^H\mathbf{h} \geq 1 & (6) \\ & \mathbf{w}^H(-\mathbf{h}) \leq -1 & (7) \end{aligned}$$

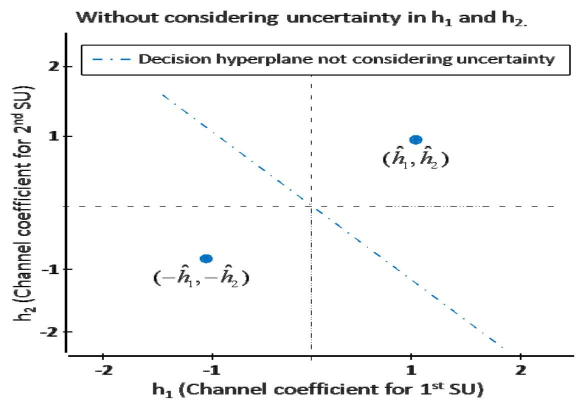

Fig. 1. Plot of uncertainty in channel coefficient of SU, with N=2, $A = \mathcal{D}(1, 0.5)$, h_1 and $h_2 \in \mathcal{R}$ are Rayleigh distributed.

By concatenating the real and imaginary parts of the vectors $\mathbf{w}, \mathbf{h}$, it can be readily seen that the above optimization problem is convex, as the L_2 norm function $\|\cdot\|_2$ is convex and the inequality constraints are affine. Note that the linear constraints in (6) and (7) are essentially the same. However, we deliberately retain this redundant structure to allow possible generalization to detection scenarios where the beacon signals are not necessarily antipodal in nature. Further, it can be readily seen that the matched filter $\mathbf{w} \triangleq \frac{1}{\|\mathbf{h}\|}\mathbf{h}$ is the unique minimizer of the above constrained convex cost function. We present a novel framework next to compute a robust optimal decision rule in the presence of CSI uncertainty in cooperative spectrum sensing scenarios.

III. Robust Detector with CSI Uncertainty

As described previously, obtaining accurate channel state information regarding $\mathbf{h}$ is challenging owing to several factors such as fading, estimation error from receiver noise, limited feedback in wireless channels etc. Hence, very frequently, only a nominal estimate $\hat{\mathbf{h}}$ of the true channel coefficient vector $\mathbf{h}$ is available at the fusion center. This possible variation in channel estimate with respect to the true channel coefficient vector $\mathbf{h}$ can be described by the following uncertainty set [8] paradigm as,

$$\mathbf{h} \in \left\{\hat{\mathbf{h}} + \mathbf{A}\mathbf{u} \mid \|\mathbf{u}\| \leq 1\right\} \quad (8)$$

Thus, the unknown true channel coefficient vector $\mathbf{h}$ lies in an uncertainty ball in (NN_r) dimensional space, where the vector $\mathbf{u} \in \mathbb{C}^{NN_r \times 1}$ is such that $\|\mathbf{u}\|_2 \leq 1$. The matrix $\mathbf{A} \in \mathbb{C}^{NN_r \times NN_r}$ describes statistical variations in $\mathbf{h}$. Thus $\mathbf{h}$ in (8) represents a vector lying inside the ellipsoid with center $\hat{\mathbf{h}}$.

Hence, in the above scenario with uncertainty, the optimal robust detector maximizes the worst case distance between the ellipsoidal uncertainty sets. For example, consider the scenario shown in Fig.1 with $N = 2$ SU with each having one transmit antenna and one receive antenna, i.e., $N_t = 1$ and $N_r = 1$. In the case of no uncertainty, the estimate of the channel coefficient $\left(\hat{h}_1, \hat{h}_2\right)$ coincides with the exact

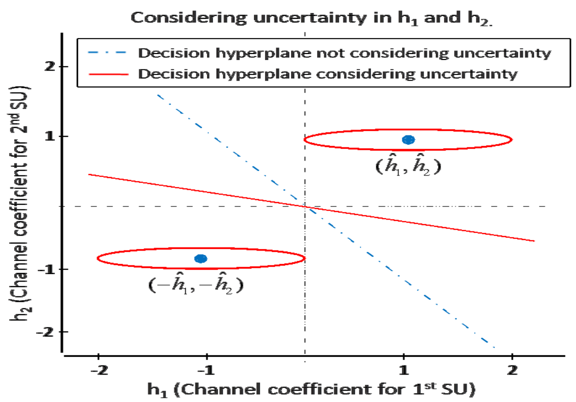

Fig. 2. Plot of uncertainty in channel coefficient of SU, with N=2, $A = \mathcal{D}(1, 0.25)$, h_1 and $h_2 \in \mathcal{R}$ are Rayleigh distributed.

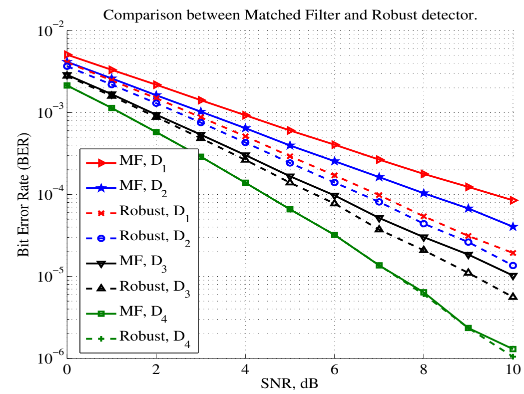

Fig. 3. Comparison between Matched Filter and Robust detector for $N_r = 2$, $N_t = 2$ MIMO, $N = 4$, $\mathbf{D}_1 = \mathcal{D}(1.6, 1.4, 1.2, 1)$, $\mathbf{D}_2 = \mathcal{D}(1.28, 1.12, 0.98, 0.8)$, $\mathbf{D}_3 = \mathcal{D}(0.8, 0.7, 0.6, 0.5)$ and $\mathbf{D}_4 = \mathcal{D}(0.32, 0.28, 0.24, 0.2)$.

channel coefficients (h_1, h_2). Let $\left(-\hat{h}_1, -\hat{h}_2\right)$ correspond to hypothesis $\mathcal{H}_0$ and denote the absence of PU and $\left(\hat{h}_1, \hat{h}_2\right)$ correspond to hypothesis $\mathcal{H}_1$ and denote the presence of PU in (2) and (3) respectively for the two SU. As per the discussion above, in the absence of uncertainty, the decision hyperplane that minimizes the detection error is the hyperplane that bisects the line joining the two antipodal hypothesis points described above. This is shown in Fig.1.

Now consider the scenario illustrated in Fig.2, where the true channel coefficients h_1, h_2 lie in the ellipsoidal uncertainty set centered at the nominal channel estimate $\left(\hat{h}_1, \hat{h}_2\right)$. Hence, the hypothesis points $\mathcal{H}_1$, $\mathcal{H}_0$ are in turn the ellipsoidal sets centered at $\left(\hat{h}_1, \hat{h}_2\right)$, $\left(-\hat{h}_1, -\hat{h}_2\right)$ respectively. Further, for illustration, the uncertainty covariance considered therein is $\mathbf{A} = \mathcal{D}(1, 0.25)$, where $\mathcal{D}$ denotes a diagonal matrix. The estimate of h_2 has a greater reliability compared to that of h_1. Naturally then, the optimal decision hyperplane is one which maximally separates these uncertainty sets corresponding to the hypothesis points. This optimal hyperplane is shown in the figure. Observe that it progressively places lower emphasis on the report of user 1. Asymptotically, in the limit where the uncertainty in h_1 tends to ∞, the uncertainty ellipsoid becomes a degenerate line parallel to the x-axis, thus entirely making a decision based on the report of user 2.

Hence, the optimal decision hyperplane which maximizes the distance between the ellipsoidal uncertainty sets can be computed as,

$$\begin{aligned} \min. \quad & \|\mathbf{w}\|_2 \\ \text{s.t.} \quad & \min_{\|\mathbf{u}\| \leq 1} \mathbf{w}^H \left(\hat{\mathbf{h}} + \mathbf{A}\mathbf{u}\right) \geq 1 \qquad (9) \\ & \max_{\|\mathbf{u}\| \leq 1} \mathbf{w}^H \left(-\mathbf{h} + \mathbf{A}\mathbf{u}\right) \leq -1 \qquad (10) \end{aligned}$$

Thus, the constraint $\min_{\|\mathbf{u}\| \leq 1} \mathbf{w}^H \left(\hat{\mathbf{h}} + \mathbf{A}\mathbf{u}\right)$ represents the worst case ellipsoidal distance, and maximizing the worst case distance between the uncertainty ellipsoids leads to a robust classifier. However, the constraint above can be equivalently represented as,

$$\begin{aligned} \min_{\|\mathbf{u}\| \leq 1} \mathbf{w}^H \left(\hat{\mathbf{h}} + \mathbf{A}\mathbf{u}\right) &= \mathbf{w}^H \hat{\mathbf{h}} + \min_{\|\mathbf{u}\| \leq 1} \mathbf{w}^H \mathbf{A}\mathbf{u} \\ &= \mathbf{w}^H \hat{\mathbf{h}} + \mathbf{w}^H \mathbf{A} \left(-\frac{\mathbf{A}^H \mathbf{w}}{\|\mathbf{A}^H \mathbf{w}\|}\right) \\ &= \mathbf{w}^H \hat{\mathbf{h}} - \left\|\mathbf{A}^H \mathbf{w}\right\| \end{aligned}$$

where the second equality above follows from the fact that the minimum of $\mathbf{w}^H \mathbf{A}\mathbf{u}$ for $\|\mathbf{u}\| \leq 1$ occurs when $\mathbf{u} = -\frac{\mathbf{A}^H \mathbf{w}}{\|\mathbf{A}^H \mathbf{w}\|}$. Hence, the optimization paradigm for construction of the robust detector above can be equivalently formulated as,

$$\begin{aligned} \min. \quad & \|\mathbf{w}\|_2 \\ \text{s.t.} \quad & \mathbf{w}^H \hat{\mathbf{h}} - \left\|\mathbf{A}^H \mathbf{w}\right\| \geq 1 \\ & -\mathbf{w}^H \hat{\mathbf{h}} + \left\|\mathbf{A}^H \mathbf{w}\right\| \leq -1 \end{aligned} \qquad (11)$$

The above problem can readily seen to be a SOCP optimization problem [8]. Moreover, it is convex and can be readily solved by a conic solver [9] to yield the robust detector minimizing the worst case error for cooperative spectrum sensing.

A. *Relaxed Robust and Multicriterion Detection*

At low SNR and deep fade conditions, the two hypothesis ellipsoids are not always guaranteed to be strictly separated. Hence, we formulate a relaxed robust discrimination (RRD) version of the above problem as,

$$\begin{aligned} \min. \quad & b \\ \text{s.t.} \quad & \mathbf{w}^H \hat{\mathbf{h}} - \left\|\mathbf{A}^H \mathbf{w}\right\| \geq 1 - b \\ & -\mathbf{w}^H \hat{\mathbf{h}} + \left\|\mathbf{A}^H \mathbf{w}\right\| \leq -1 + b \\ & b \geq 0. \end{aligned} \qquad (12)$$

where $b \geq 0$, the non-negative slack variable, is a measure of the constraint violation. The above relaxed formulation maximally separates the hypothesis ellipsoids while minimizing the number of misclassified points [8]. This SOCP problem yields

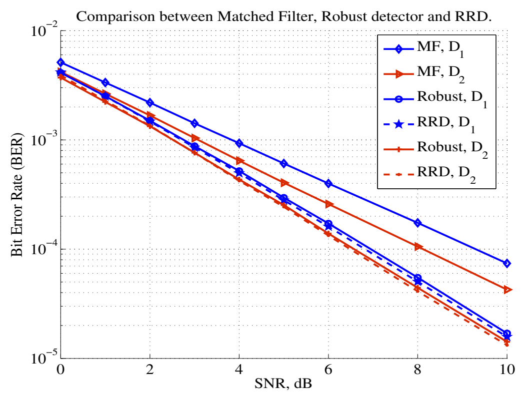

Fig. 4. Comparison between Matched Filter, Robust detector and RRD for MIMO with $N_r = 2$, $N_t = 2$, $N = 2$, $\mathbf{D}_1 = \mathcal{D}(1.6, 1.4, 1.2, 1)$ and $\mathbf{D}_2 = \mathcal{D}(1.28, 1.12, 0.98, 0.8)$.

the relaxed optimal detector for cooperative spectrum sensing applications. A multicriterion robust detector (MRD), which is a trade-off between the robust detector (11) and RRD (12) can be formulated by introducing a non-negative weighting parameter λ as,

$$\begin{aligned} \min.\ & \|\mathbf{w}\|_2 + \lambda b \\ \text{s.t. } & \mathbf{w}^H\hat{\mathbf{h}} - \left\|\mathbf{A}^H\mathbf{w}\right\| \geq 1 - b \\ & -\mathbf{w}^H\hat{\mathbf{h}} + \left\|\mathbf{A}^H\mathbf{w}\right\| \leq -1 + b \\ & b \geq 0. \end{aligned} \tag{13}$$

Below we present simulation results to validate the performance of the PU sensing schemes described above.

IV. Simulation Results

We consider a 2×2 MIMO scenario with $N = 2$ SUs, each having $N_r = 2$ receive antennas. The PU base station possesses $N_t = 2$ transmit antennas. We consider four different levels of CSI uncertainty, characterized by the uncertainty matrices $\mathbf{A}_i$ such that $\mathbf{A}_i = \mathbf{U}\mathbf{D}_i\mathbf{U}^T$, where $\mathbf{U}$ is a random unitary matrix, and the diagonal matrices $\mathbf{D}_1 = \mathcal{D}(1.6, 1.4, 1.2, 1)$, $\mathbf{D}_2 = \mathcal{D}(1.28, 1.12, 0.98, 0.8)$, $\mathbf{D}_3 = \mathcal{D}(0.8, 0.7, 0.6, 0.5)$ and $\mathbf{D}_4 = \mathcal{D}(0.32, 0.28, 0.24, 0.2)$ respectively. We begin by comparing the detection error performance of the robust spectrum sensing scheme (11) with that of the conventional nominal channel estimate based detector, for each uncertainty scenario in Fig.3. It can be seen from the results that the robust detector significantly outperforms the conventional detector and further the performance gap widens as uncertainty increases.

For the detection scenario in Fig.4 we consider random statistical variation matrices $\mathbf{A}_i$ corresponding to the diagonal matrices $\mathbf{D}_1$ and $\mathbf{D}_2$ respectively. We present the performance of the RRD based spectrum sensing scheme described in section III-A and compare its performance with the robust detector and the conventional matched filter detector. Fig.4 follows the trend of Fig. 3 and it can also be seen that the RRD has a slightly superior performance compared to the robust detector. In Fig.5 we plot the spectrum sensing performance of the MRD scheme. It is evident from therein that the performance of the MRD is similar to that of the robust detector and it outperforms the conventional MF detector.

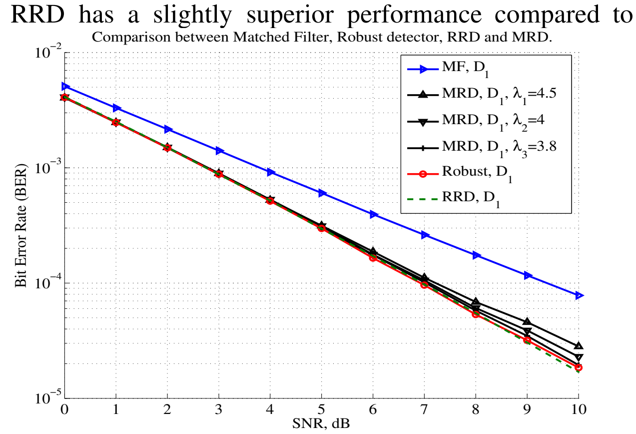

Fig. 5. Comparison between Matched Filter, Robust detector, RRD and MRD ($\lambda_1 = 4.5$, $\lambda_2 = 4$, $\lambda_3 = 3.8$) for MIMO with $N_r = 2$, $N_t = 2$, $N = 2$ and $\mathbf{D}_1 = \mathcal{D}(1.6, 1.4, 1.2, 1)$.

V. Conclusion

In this paper we presented novel techniques for cooperative spectrum sensing in a CR network. We proposed a robust detector for cooperative primary user detection which considers the channel uncertainty. We further demonstrated that the worst case detection error minimization can be formulated as a SOCP. It was observed that the performance of the robust detector and the allied RRD and MRD schemes is superior compared to that of the conventional uncertainty agnostic detector for spectrum sensing.

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